CH 11: Balanced Three-phase Circuit.

three sources — Three-phase $\equiv$ 3-ϕ

single-phase $\equiv$ 1-ϕ

Balanced 3-ϕ Voltages:

They are 3 voltage sources that have identical amplitudes and frequency, but are out of phase with each other by exactly 120°

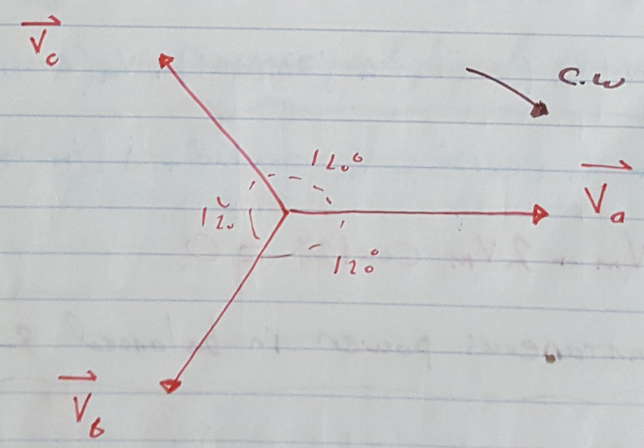
sequences of balanced 3-ϕ systems

1) Positive sequence "abc sequence"

$v_a(t) = V_m \cos(\omega t + \theta_v)$, $\vec{V_a} = V_m \angle \theta_v$

$v_b(t) = V_m \cos(\omega t + \theta_v - 120^\circ)$, $\vec{V_b} = V_m \angle \theta_v - 120^\circ$

$v_c(t) = V_m \cos(\omega t + \theta_v + 120^\circ)$, $\vec{V_c} = V_m \angle \theta_v + 120^\circ$

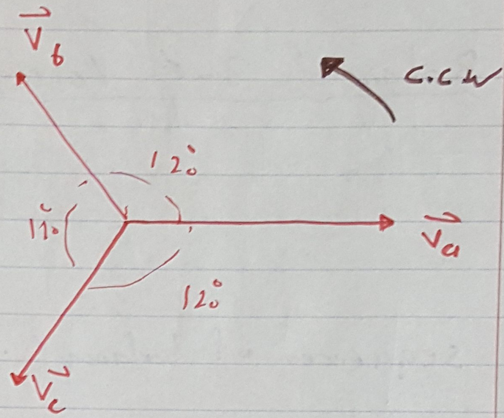


2) Negative sequance "acb sequence"

$\vec{V}_a = V_m \angle \theta_v$

$\vec{V}_b = V_m \angle \theta_v + 120^\circ$

$\vec{V}_c = V_m \angle \theta_v - 120^\circ$



Balanced 3-ϕ Voltages:

$\boxed{\theta_v = 0}$

$v_a(t) + v_b(t) + v_c(t)$ t-domain

$\vec{V}_a + \vec{V}_b + \vec{V}_c$ phaser-domain

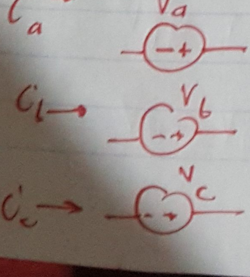
$= V_m \angle 0^\circ + V_m \angle -120^\circ + V_m \angle 120^\circ$

$= V_m + V_m(\cos 120 + j\sin(-120)) + V_m(\cos(120) + j\sin(120^\circ))$

$= V_m + V_m \cos 120^\circ + V_m \cos 120^\circ + j\underbrace{(V_m \sin(120) - V_m \sin(120))}_{}^{0}$

$= V_m + 2V_m \cos 120^\circ = 0$

Instantaneous power in balanced 3-ϕ systems



$p(t) = p_a(t) + p_b(t) + p_c(t)$

Total instantaneous power

$P_a = V_a(t)\, i(t)$, $P_b = V_b(t)\, i_b(t)$, $P_c = V_c(t)\, i_c(t)$

Assume :

$V_a(t) = V_m \cos(\omega t + \theta_v)$, $i_a = I_m \cos(\omega t + \theta_i)$

$V_b(t) = V_m \cos(\omega t + \theta_v - \frac{2\pi}{3})$ $\quad i_b = I_m \cos(\omega t + \theta_i - \frac{2\pi}{3})$

$V_c(t) = V_m \cos(\omega t + \theta_v + \frac{2\pi}{3})$ $\quad i_c = I_m \cos(\omega t + \theta_i + \frac{2\pi}{3})$

$P(t) = V_a(t)\, i_a(t) + V_b(t)\, i_b(t) + V_c(t)\, i_c(t)$

$\underset{3\text{-}\phi}{P(t)} = \frac{3}{2} V_m I_m \cos(\theta_v - \theta_i) =$ Average power

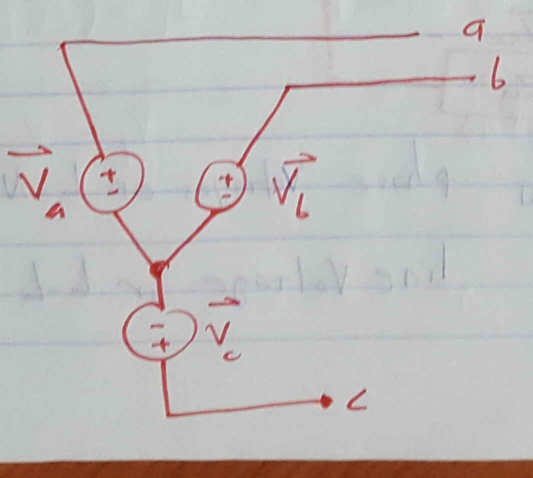
(No double frequency component)

instantaneous power

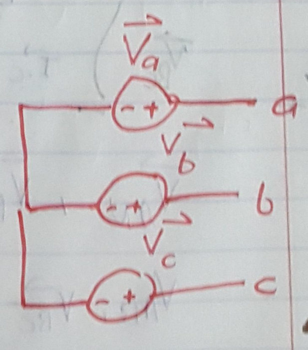
$\underset{1\text{-}\phi}{p(t)} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + f(2\omega t)$

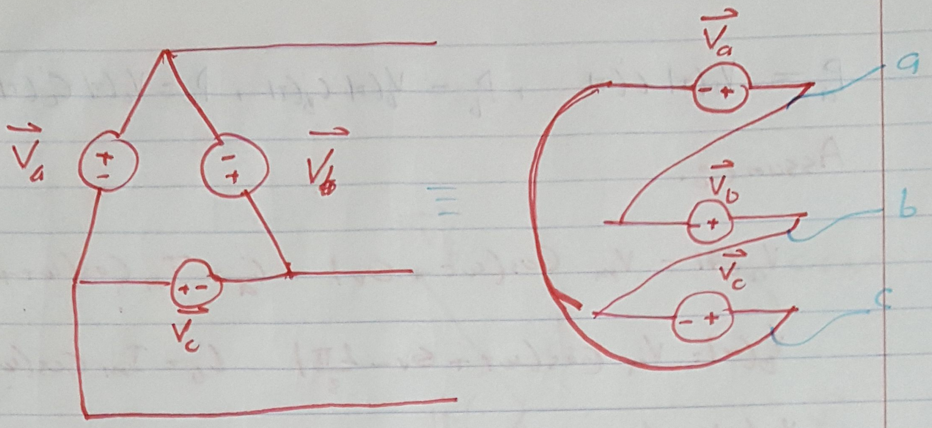
Source and Load Connections.

Source Y on Δ



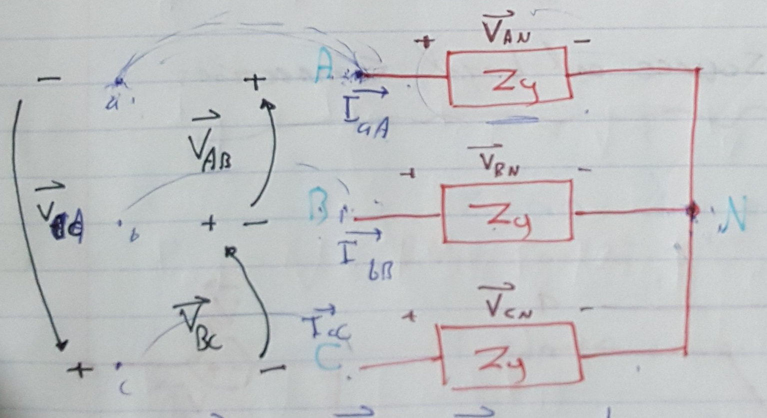
Y





Source	Load
Y	Y
Y	Δ
Δ	Y
Δ	Δ

line and phase Voltages in Y-connected load or Source.



in this case $\vec{I}_L = \vec{I}_\phi$

$\vec{V}_{AN}$, $\vec{V}_{BN}$, $\vec{V}_{CN}$ phase Voltages or L-N Voltages

$\vec{V}_{AB}$, $\vec{V}_{BC}$, $\vec{V}_{CA}$ line Voltage or L-L Voltages

$\vec{I}_{aA}$, $\vec{I}_{bB}$, $\vec{I}_{cC}$ line Currents.

Assume :

$$\vec{V}_{AN} = V_\phi \angle \theta_{v\phi}$$

$$\vec{V}_{BN} = V_\phi \angle \theta_{v\phi} - \frac{2\pi}{3}$$

$$\vec{V}_{CN} = V_\phi \angle \theta_{v\phi} + \frac{2\pi}{3}$$

$$\vec{V}_{AB} = \vec{V}_{AN} - \vec{V}_{BN} = V_\phi \angle \theta_{v\phi} - V_\phi \angle \theta_{v\phi} - \frac{2\pi}{3}$$

$$\vec{V}_{AB} = \left(V_\phi \cos(\theta_{v\phi})\right) + j V_\phi \sin(\theta_{v\phi})$$

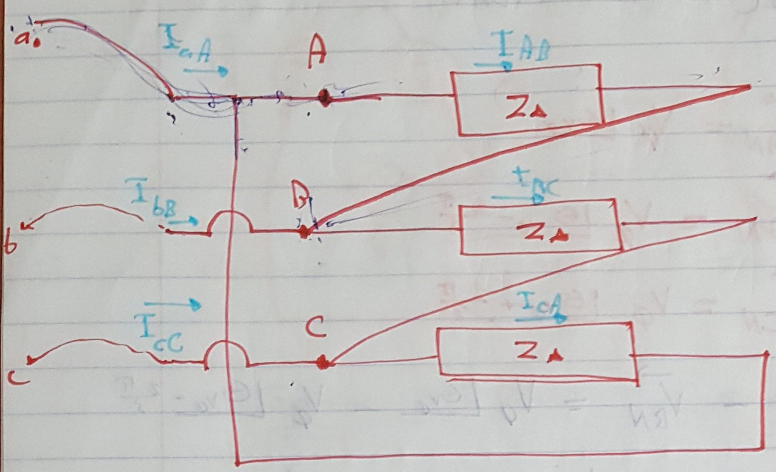
$$- \left(V_\phi \cos\left(\theta_{v\phi} - \frac{2\pi}{3}\right) + j V_\phi \sin\left(\theta_{v} - \frac{2\pi}{3}\right)\right)$$

$$\boxed{\vec{V}_{AB} = \sqrt{3}\, V_\phi \angle \theta_{v\phi} + 30^\circ}$$

$$\vec{V}_{AB} = \left(\sqrt{3} \angle 30^\circ\right) \vec{V}_{AN}$$

$$\vec{V}_{LL} = \left(\sqrt{3} \angle 30^\circ\right) \cancel{\text{ } } \vec{V}_{L-N}$$

Δ _ Connected Load on source



I_{AB} , I_{BC} , I_{CA} $\Rightarrow$ phase current

I_{aA} , I_{bB} , I_{cC} $\Rightarrow$ line current

V_{AB} , V_{BC} , V_{CA} Phase voltages

$$\underbrace{\vec{I}_{AB} = I_\phi \angle \theta_{i\phi}}, \quad \underbrace{\vec{I}_{BC} = I_\phi \angle \theta_{i\phi} - \tfrac{2\pi}{3}}, \quad \underbrace{\vec{I}_{CA} = I_\phi \angle \theta_{i\phi} + \tfrac{2\pi}{3}}$$

$$\underbrace{I_{aA} = \vec{I}_{AB} - \vec{I}_{CA} = \sqrt{3}\, I_\phi \angle \theta_{i\phi} - 30^\circ}$$

$$I_{aA} = (\sqrt{3} \angle 30^\circ)(I_\phi \angle \theta_{i\phi}) \quad , \quad \vec{I}_L = (\sqrt{3} \angle -30^\circ)\, \vec{I}_\phi$$

$$I_{aA} = (\sqrt{3} \angle -30^\circ)\, \vec{I}_{AB} \qquad \vec{V}_L = \vec{V}_\phi$$

Y-connected load or source

$$V_L = (\sqrt{3}\angle 30^\circ)\,\vec{V_\phi}$$

$$\vec{I_L} = \vec{I_\phi}$$

Δ-Connected load or source

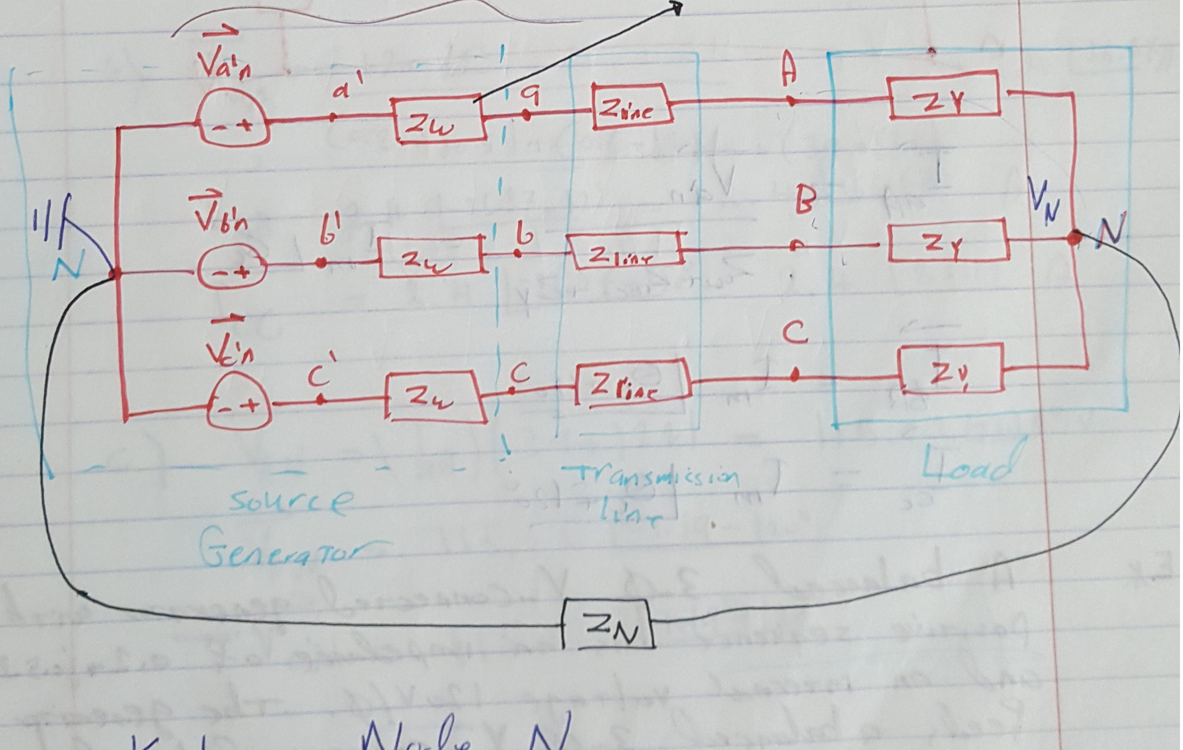
$$\vec{V_L} = \vec{V_\phi}$$

$$\vec{I_L} = (\sqrt{3}\angle -30^\circ)\,\vec{I_\phi}$$

Y-Y Analysis

source   load

winding impedance



Kcl at Node N

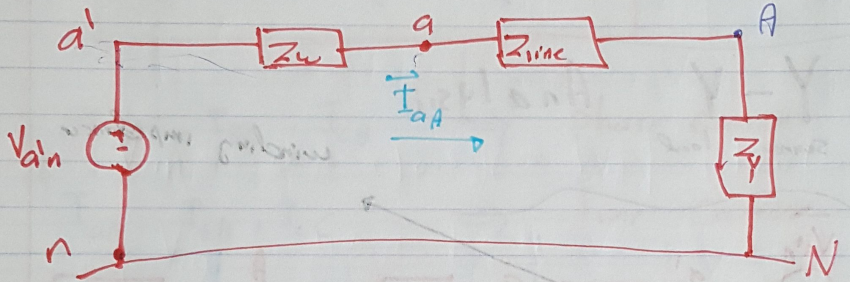
$$\frac{\vec{V_N} - \vec{V_{a'N}}}{Z\phi} + \frac{\vec{V_N} - \vec{V_{b'n}}}{Z\phi} + \frac{\vec{V_N} - \vec{V_{c'n}}}{Z\phi} + \frac{\vec{V_N}}{Z_N} = 0$$

$$\frac{3}{Z_\phi}\vec{V_N} - \frac{1}{Z_\phi}\left(\vec{V_{a'n}} + \cancel{\vec{V_{b'n}} + \vec{V_{c'n}}}\right) + \frac{V_N}{Z_N} = 0$$

$$\left(\frac{3}{Z_\phi} + \frac{1}{Z_N}\right)\vec{V_N} = 0$$

$$\boxed{\vec{V_N} = 0} \Rightarrow \boxed{I_N = 0}$$

$1-\phi$ equivalent Circuit



$$\vec{I_{aA}} = \frac{\vec{V_{a'n}}}{Z_w + Z_{line} + Z_Y} = I_m \angle\theta_c$$

$$\vec{I_{bB}} = I_m \angle\theta_c \mp 120^\circ$$

$$\vec{I_{cC}} = I_m \angle\theta_c \pm 120^\circ$$

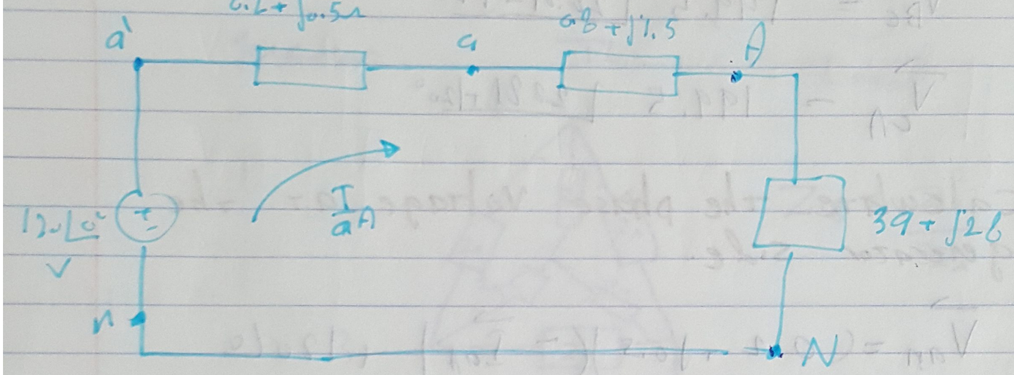
Ex A balanced 3-ϕ Y-connected generator with positive sequence has an impedance of $0.2 + j0.5\ \Omega/\phi$ and an internal voltage $120\,V/\phi$. The generator feeds a balanced 3-ϕ Y-connected Load having an impedance of $39 + j28\ \Omega/\phi$, The impedance of TL is $0.8 + j1.5\ \Omega/\phi$

a) Draw the 1-ϕ equivalent circuit

b) Calculate the line current

c) calculate the phase voltage at the load side.

d) " " line voltages " " " "



b) $\vec{I}_{aA} = \dfrac{120 \angle 0°}{(0.2 + j0.5) + (0.8 + j1.5) + (39 + j28)} = 2.4 \angle -36.87 A$

$I_{bB} = 2.4 \angle -36.87 - 120° = 2.4 \angle 156.87° A$

$\vec{I}_{cC} = 2.4 \angle -36.87 + 120° = 2.4 \angle 83.13 A$

c) $\vec{V}_{AN} = (\vec{I}_{aA})(39 + j28) = 115.22 \angle -1.19° V$

$\vec{V}_{BN} = 115.22 \angle -1.19 - 120°$

$\vec{V}_{CN} = 115.22 \angle -1.19° + 120°$

d) $\vec{V}_{AB} = \sqrt{3}\angle 30^\circ \, (\vec{V}_{AN})$

$\vec{V}_{AB} = (\sqrt{3}\angle 30^\circ)(115.22\angle -1.19^\circ)$ V

$= 199.5\angle 28.81^\circ$

$\vec{V}_{BC} = 199.5\angle 28.81 - 120^\circ$

$\vec{V}_{CA} = 199.5\angle 28.81 + 120^\circ$

E Calculate the phase voltage at the generator side.

$\vec{V}_{an} = (0.2 + j0.5)(-\vec{I}_{aA}) + 120\angle 0$

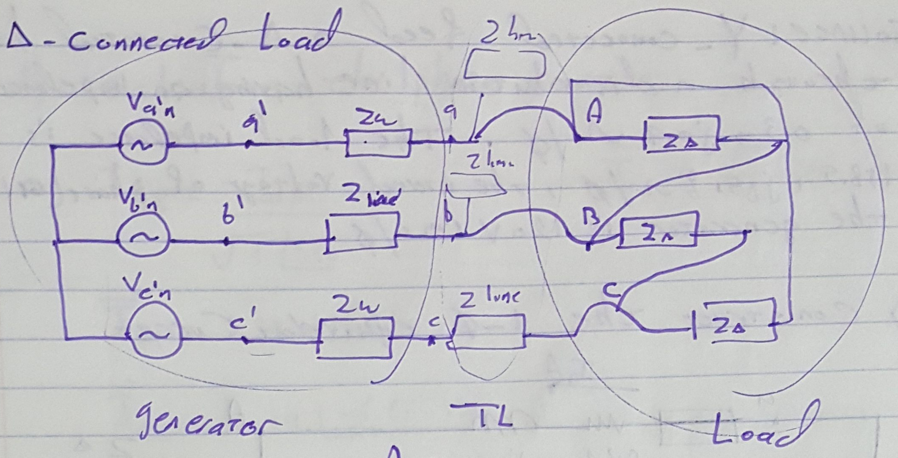
$\vec{V}_{aN} = 118.9\angle -0.32^\circ$

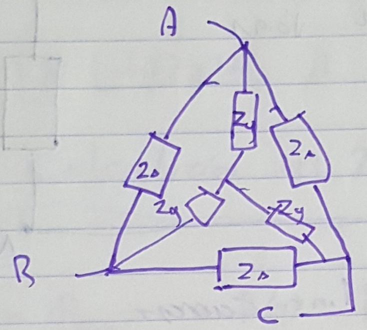
$\vec{V}_{bn} = 118.9\angle -0.32 - 120^\circ$

$\vec{V}_{cn} = 118.9\angle -0.32 + 120^\circ$

$Y \rightarrow \Delta$

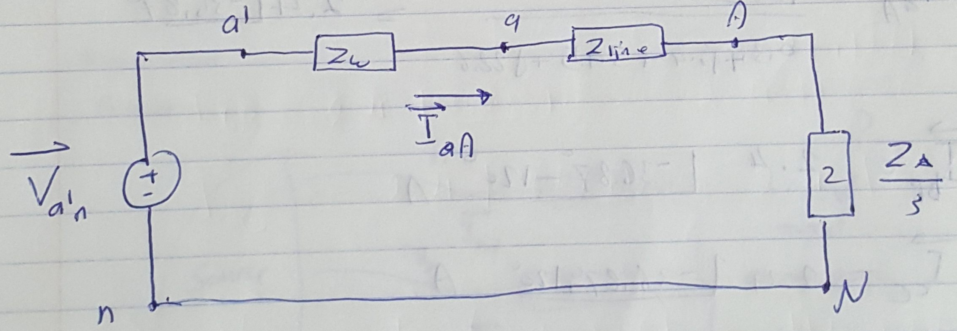
Δ-Connected Load





$$Z_y = \frac{Z_\Delta^2}{3Z_\Delta} = \frac{Z_\Delta}{3}$$

1-ϕ equivalent circuit



$$\vec{I}_{aA} = \frac{\vec{V}_{a'n}}{Z_w + Z_{line} + \frac{Z_\Delta}{3}}$$

$\vec{I}_L = (\sqrt{3} \angle -30^\circ) \vec{I}_\phi$

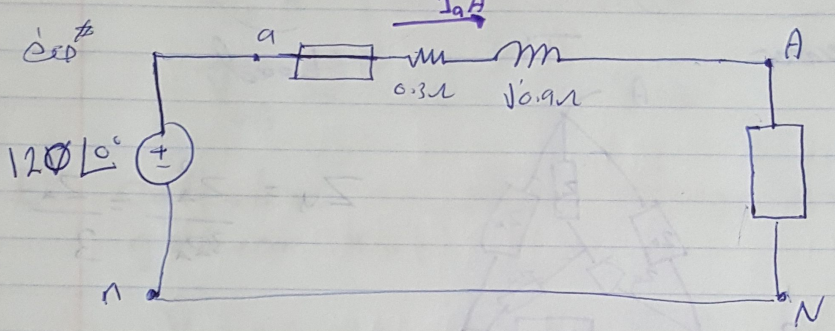
$\vec{V}_L = \vec{V}_\phi$

Δ ← Y

positive sequance.

source: Y - connected feed a Δ - connected load through a distribution line having an impedance of 0.3 + j0.9 Ω/φ. The load impedance is 118.5 + j85.8 Ω/φ, the internal voltage of phase (a) of the generator is 120 V rms/φ

a) construct the 1-φ equivelant circut



$$\frac{Z_\Delta}{3}$$

$$\frac{118.5}{3} + \frac{j85.8}{3}$$

$$= 39.5 + j28.6$$

b) calculate the line current

$$\vec{I}_{aA} = \frac{120\angle 0}{0.3 + j0.9 + 39.5 + j28.6} = 2.4\angle -36.87^{\circ}$$

$$\vec{I}_{bB} = 2.4\angle -36.87^{\circ} - 120^{\circ}\ A$$

$$\vec{I}_{cC} = 2.4\angle -36.87^{\circ} + 120^{\circ}\ A$$

here

c) Calculate the phase voltages at the load sides.

$$V_{AN} = \left(\frac{Z_\Delta}{3}\right)\left(\vec{I}_{aA}\right) = 114.04\angle -0.96^{\circ}$$

$$V_{AB} = (+\sqrt{3}\angle 30^{\circ})\ \vec{V}_{AN} = 202.72\angle 29.04\ V$$

$$\vec{V}_{BC} = 202.72\angle 29.04 - 120^{\circ}$$

$$\vec{V}_{CA} = 202.72\angle 29.04 + 120^{\circ}$$

في Δ تكون
نفس الجهد
phase
و line
نفس

$\boxed{AB}$

(d) calculate the phase current.

$$I_{AB} = \frac{I_{aA}}{\sqrt{3}\angle -30^\circ} \qquad \vec{I}_L = \sqrt{3}\angle -30^\circ \; \vec{I}_\phi$$

$$\vec{I}_{AB} = 1.39 \angle -6.87^\circ \text{ A}$$

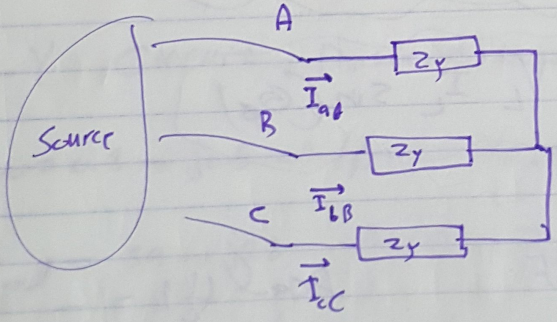
$$\vec{I}_{BC} = 1.39 \angle -6.87^\circ - 120^\circ \text{ A}$$

$$\vec{I}_{CA} = 1.39 \angle -6.87 + 120^\circ \text{ A}$$

Power calculation in $3-\phi$ circuit.

$$P_{3-\phi}(t) = P_{avg} \quad , \quad P_{1-\phi}(t) = P_{avg} + f(2wt)$$

Instantaneous Powers



$$P_T = 3 P_\phi$$

Total average power

Power of each phase

$$P_\phi = V_\phi I_\phi \cos(\theta_\phi)$$

Power factor angle

$$P_a = V_{AN} \cos(\theta_{V_{AN}} - \theta_{I_{aA}})$$

$$P_b = V_{BN} I_{bB} \cos(\theta_{V_{BN}} - \theta_{I_{bB}})$$

$$P_c = V_{CN} I_{cC} \cos(\theta_{V_{CN}} - \theta_{I_{cC}})$$

$V_{AN} = V_{BN} = V_{CN} = V_\phi$

$I_{aA} = I_{bB} = I_{cC} = I_\phi$

$\theta_{V_{AN}} - \theta_{I_{aA}} = \theta_{V_{BN}} - \theta_{I_{bB}} = \theta_{V_{CN}} - \theta_{I_{cC}} = \theta_\phi$

$P_a = P_b = P_c = P_\phi$

$P_T = 3 V_\phi I_\phi \cos(\theta_\phi)$

$V_L = \sqrt{3} V_\phi$

$I_L = I_\phi$

$P_T = 3\left(\frac{V_L}{\sqrt{3}}\right)(I_L)\cos(\theta_\phi)$

$P_T = \sqrt{3} V_L I_L \cos(\theta_\phi)$

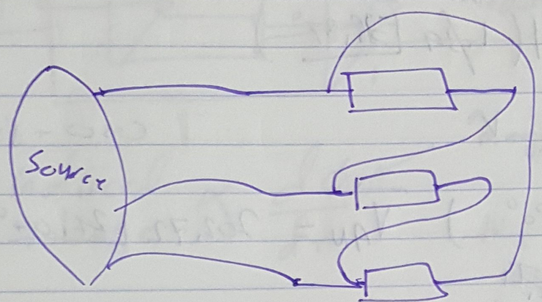
$Q_T = \sqrt{3} V_L I_L \sin(\theta_\phi)$

Ex $P = \frac{3}{2} V I pF$ Peal $(\frac{1}{2})$ - -

$P = \frac{\sqrt{3}}{2} V I PF$, peal $(\frac{1}{2})$ -

$\vec{S} = P_T + jQ_T$ $\quad \theta_{V\phi} - \theta_{i\phi}$

$\vec{S} = \sqrt{3}\, V_L\, I_L \angle \theta_\phi$



$P_T = 3 V_\phi I_\phi \cos(\theta_\phi)$

$V_L = V_\phi \qquad I_L = \sqrt{3}\, I_\phi$

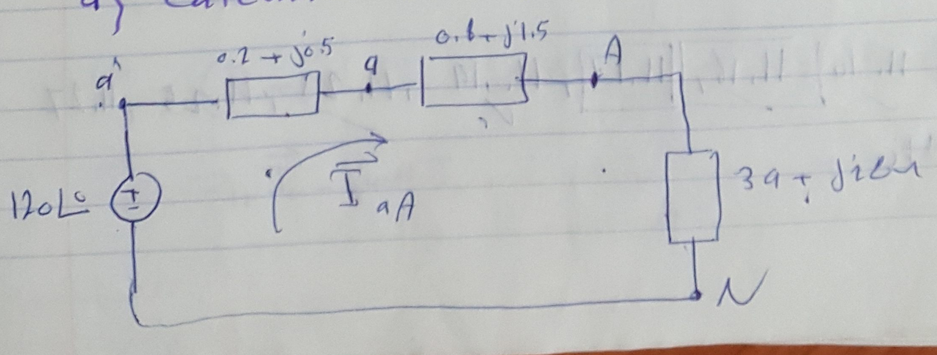
$P_T = 3 V_L \dfrac{I_L}{\sqrt{3}} \cos(\theta_\phi) = \sqrt{3}\, V_L I_L \cos(\theta_\phi)$

Ex: Y-Y circuit, positive sequence generator, Z_w

$Z_w = 0.2 + j0.5\ \Omega/\phi$, $\quad V_{a'n} = 120\ V/\phi$

$Z_{Load} = 39 + j26\ \Omega/\phi$, $\quad Z_{line} = 0.8 + j1.5\ \Omega/\phi$

a) calculate the total power delivered to the load.



$e^{i\theta} = c$

$$I_{aA} = \frac{120\angle 0°}{40+j30} = \cancel{1.24}\ 2.4 \angle -6.87° \text{ A}$$

$I_L = 1.39$

$$\vec{V}_{AN} = (39+j28)(\cancel{1.39}\ 2.4 \angle \cancel{36.87°})$$

$$\vec{V}_{AN} = 115.22\angle 1.19 \text{ V}$$

$\cos\theta + j\sin\theta$

$$V_L = (\sqrt{3}\angle 30°)\ \vec{V}_{AN} = 202.72\angle 29.04° \text{ V}$$

$$P_T = \sqrt{3}\,(\cancel{1.39}\ 2.4)(202.72)\cos(-1.19° - 6.87)$$

$$P_T = 672.98 \text{ W}$$

b) Calculate the total reactive and complex power delivered to the Load

$$Q_T = \sqrt{3}\,(1.39)(202.72)\sin(-1.19° + 6.87°)$$

$$S_T = P_T + jQ_T \quad \text{—} \quad 2\ 162.$$

c) Calculate the total avg Power lost in the TL

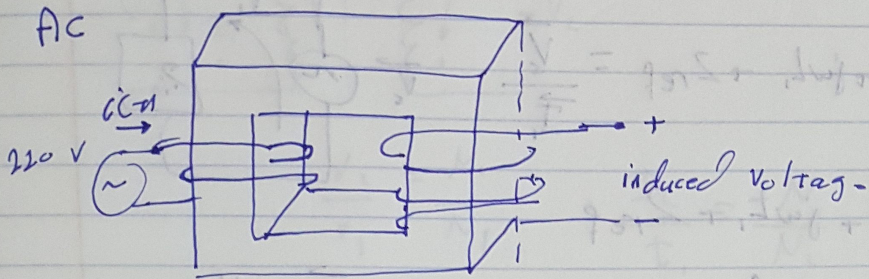
$$P_{line} = 3\, I_{aA}^2\,(0.2) = (3)(2.4)^2(0.8) = 13.824 \text{ W}$$

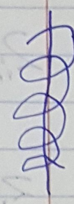
Problem to solve. 11.9, 11.10, $\boxed{11.16,}$ $\boxed{11.17}$, 11.19, 11.23, 11.25, 11.27

11.40.

Transformer

AC



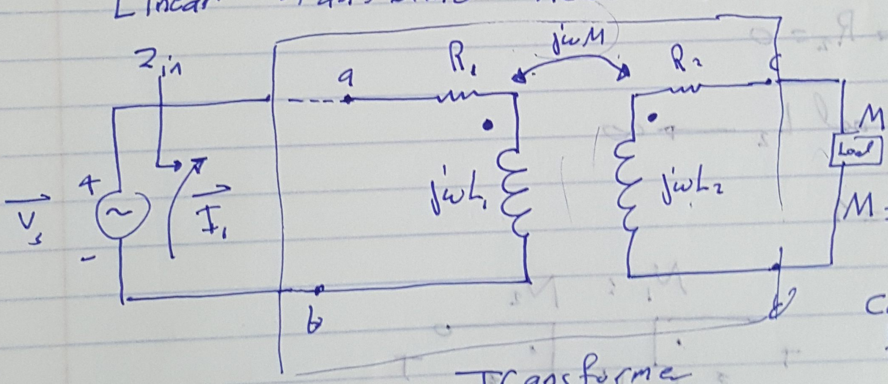


induced voltage $\propto \frac{d\phi}{dt}$ ← "magnetic flux"

N_2

$V_s \propto \frac{d\phi}{dt}$, N_1

Linear Transformer "Real Transformer"



Transforme

M: mutual coupling

$M = K\sqrt{L_1 L_2}$ "ch6"

↓

Coupling factor

KVL in loop 1

$\vec{V}_s = (R_1 + j\omega L_1)\vec{I}_1 - j\omega M \vec{I}_2$

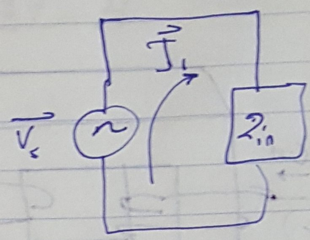
$\vec{V}_s = Z_{11}\vec{I}_1 - j\omega M \vec{I}_2$, $0 = Z_{22}\vec{I}_2 - j\omega M \vec{I}_1$

KVL in loop 2:

$0 = (Z_L + R_2 + j\omega L_2)\vec{I}_2 - j\omega M \vec{I}_1$

Reflected impedance

$$Z_{in} = R_1 + j\omega L_1 + Z_{ref} = \frac{\vec{V}_s}{\vec{I}_1}$$



$$Z_{in} = R_1 + j\omega L_1 + Z_{ref}$$

$$Z_{ref} = \left(\frac{\omega M}{|Z_{22}|}\right)^2 \cdot Z_{22}^*$$

$$\vec{I}_1 = \frac{V_s}{Z_{in}}$$

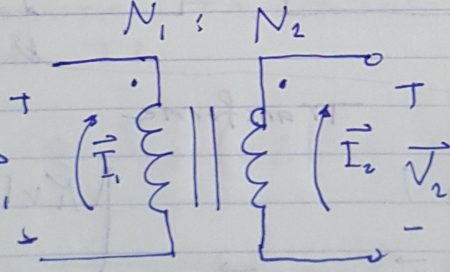
$$|Z_{22}| = |R_2 + j\omega L_2 + Z_L|$$

Ideal Transformer:

✓ $R_1 = R_2 = 0$

✓ L_1 and $L_2 \longrightarrow \infty$

✓ $K = 1$



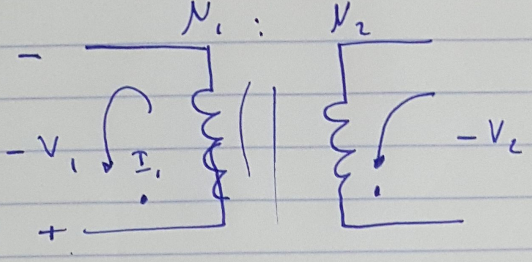
$$\frac{V_2}{V_1} = \frac{N_2}{N_1}, \quad \frac{I_2}{I_1} = \frac{N_1}{N_2}, \quad \frac{N_2}{N_1} = a$$

$\downarrow$

Transformer

Transition

Ex



$$\frac{V_2}{V_1} = \frac{N_2}{N_1} \quad , \quad \frac{I_2}{I_1} = \frac{-N_1}{N_2}$$

$$Z_{ref} = \frac{1}{a^2} Z_L \qquad a = \frac{N_2}{N_1}$$

Ex